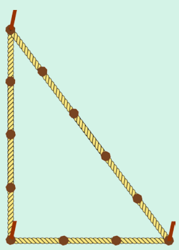
SQUARING THE CIRCLE
A PRACTICAL APPROACH

Jonathan J. Crabtree jonathanjcrabtr

For 2500 years, mathematicians had fun trying to square the circle using only a compass and straightedge. Yet in 1882 this classic Greek problem was proven to be impossible. So, what do we do?

SURVEYING ANCIENT IDEAS



Before the compass, in both India and Egypt, circles were drawn via ‘peg and rope/cord’ methods. The Egyptians, long before Pythagoras, used a rope with 12 evenly spaced knots to form right angles by making triangles with sides 3, 4 and 5. So, along with foundations for pyramids, came foundations of geo (earth) metry (measurement).

In India, from 800 BCE, when the Śulba Sūtras, (rules of cord/rope) were written, ropes were often used for altar construction. Much later, the *Hebrew Mishnat ha-Middot, (Treatise of Measures),* dated 150 CE by its translator, has; *‘The circle has three aspects: the circumference, the thread [diameter] and the roof [area]. Which is the circumference? That is the rope surrounding the circle…’* Notably, ‘line’ came to us via the Latin, ‘linea’, meaning ‘a linen thread, a string, line.’

If we use ropes, like these ancient cultures, before the Greeks added the problematic restrictions of ‘compass and straight edge’, we may find an interesting way to square the circle. So this outdoor project for students uses pegs and ‘curved edge’ (rope), like Egypt, in order to square the circle. From squaring the circle, mathematics teachers can disintegrate curves, to reveal the ancient origins of integral calculus. We will look at this in more detail in a future article.

CIRCLE SQUARING IN ANTIQUITY

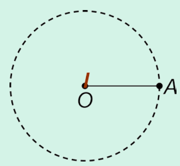
Ahmes, the scribe who wrote the Egyptian Rhind papyrus around 1650 BCE, created squares with sides equal to eight ninths the diameter of a circle. Thus, an Egyptian circle, with diameter 9 units, had an area of 63.617 square units, while the square had a greater area of 64 square units.

In India, from around 800 BCE, a solution was to create squares with sides equal to thirteen fifteenths the diameter of a circle. Thus, an Indian circle, with diameter 15 units, had an area of 176.715 square units, while the square had a lesser area of 169 square units. Edward de Bono, who coined the term lateral thinking, created *po* as an alternative to linear ‘yes/no’ thinking. Since we are presenting a lateral solution to an ancient problem, we define *po* as the ratio of a circle’s area to the area of the square derived from the circle, the Egyptian *po*, at 0.994, is better than the Indian *po*, of 1.046. Our goal is to achieve *po* = 1.

HOW TO SQUARE THE CIRCLE

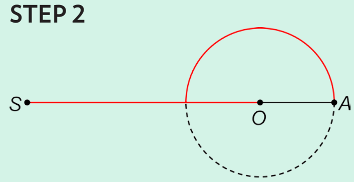
To ‘learn the ropes’ of the lost art of circle squaring, we need nothing but unmarked ropes and pegs. Our construction will be drawn directly onto the ground. We do *not* use compass and straight edge. We are, after all, star-watchers, and will not be told, four thousand years hence, that what we have done is impossible! So, to put a circle upon the ground, we place a fixed peg in the ground with an unmarked rope attached. Then, we pull a free peg at the other end, to straighten the rope as a radius. Then the free peg is swung all the way around the fixed peg, to draw a circle onto our sacred ground. Note that whenever a black dot appears in the diagrams below, it helps to hammer a peg into the ground.

STEP 1



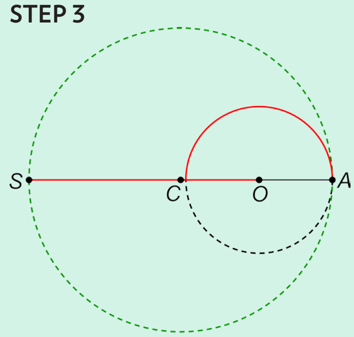
In Step 1 a pegged rope is rotated around O to draw a circle. The radius, (r), is the arbitrary unit of length OA.

STEP 2



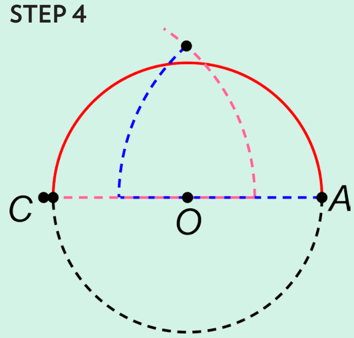
In Step 2 a rope (highlighted in red) is laid on the top half of the circumference and cut, (or just pinched and held). This rope is then extended leftward from O. We now have pegs at A, O and S.

STEP 3



In Step 3, we bisect SA at C and place a fixed peg at C. (A length of rope is easy to bisect!) Then, with a rope pegged at C, with length SC, we draw a circle around C, (shown with green dashes).

STEP 4

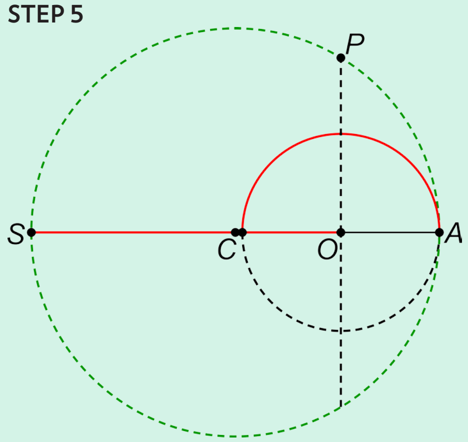


In Step 4 a perpendicular line is found from O. To do this, simply rotate the same length of rope about each end of the diameter of the circle we are squaring. The point where the two arcs intersect gives us a temporary reference point for our perpendicular line from O.

SQUARING THE CIRCLE
A PRACTICAL APPROACH (CONT.)

Jonathan J. Crabtree jonathanjcrabtr

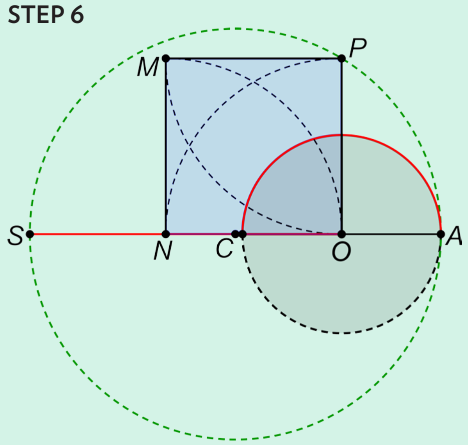
STEP 5



In Step 5 a peg point is made at P, where the perpendicular from O meets the green outer circle. A 'master rope' of length OP is created, shown as the dashed line from O to P. This is the first side of our desired square.

Your construction should resemble this (less the extra quarter circle).

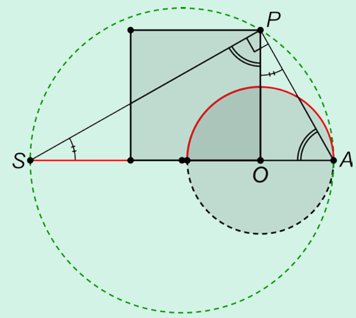
STEP 6



In Step 6, the master rope, OP, is swung around O, (anti-clockwise), to create the peg point N. Then the master rope is swung around peg point N, then peg point P, to create intersecting arcs at peg point M. Ropes pegged around $OPMN$ then produce a square with the same area as our initial circle. Behold! The circle is squared!

A PROOF, VIA EGYPT AND THALES

The Greek astronomer, Thales of Miletus, (624-547 BCE), is mainly known for his theorem that any angle inscribed in a semi-circle must be a right angle.



Triangles SOP and POA, with corresponding angles equal, are similar.

$OS : OP :: OP : OA$

Thus, $OS \times OA = OP \times OP$.

The length of OS (the semi-circumference) is πr, and OA is r, with $OP \times OP$ being the area of the square we constructed.

Area of square = $OS \times OA = \pi r \times r = \pi r^2$, the area of the circle, whatever its radius may be. And yes, our $po = 1$.

With plain rope, which can be curved in a circular arc and straightened, the circle could have been squared 2600 years ago! So, if it wasn't for the Greek, Oenopides of Chios, who in 425 BCE restricted the tools of geometric construction to straightedge and compass, mathematics would have evolved quite differently.

The author thanks The Aegean Center for the Fine Arts, from the island of Paros, where the circle was squared on Greek sand, in honour of Archimedes, author of *Measurement of a Circle* and *The Sand Reckoner*.

Left to right: A curious local child; Alexia Vlahos, USA; Spiros Mavromatis, Greece; Jane Pack (team leader), USA; Max Wolnak, USA; Maria Xu, China; Annelise Grindheim, Norway; Bea Saenger, USA; Chris Saenger with son Charlie, USA; Cari Saenger, USA; Gail Saunders, USA; Lexi Schmidt, Canada; Iriglenn Leka, Albania; Shreya Shetty, India; and Jun-Pierre Shiozawa, USA. Photograph by John Pack. www.aegeancenter.wordpress.com/2016/04/06/squaring-the-circle-take-two/.

AUTHOR

Having once failed mathematics and repeated a year at school, Jonathan Crabtree later came to enjoy exploring the evolution of mathematics and dreaming up new ideas to help explain it. Jonathan has explored hundreds of original source mathematics books and manuscripts spanning 16 languages. He welcomes your feedback.

Web: www.jonathancrabtree.com

REFERENCES AND FURTHER READING

de Bono, E. (1978). *Po: Beyond Yes and No*, pp. 9-18, Penguin, Middlesex.

Engels, H. (1977). Quadrature of the Circle in Ancient Egypt. In K. May (Ed.), *Historia Mathematica 4*(2), pp. 137-140, Springer, New York.

Gandz, S. (1932). *The Mishnat Ha Middot: The First Hebrew Geometry of About* 150 C.E., p.17, Springer.

Gillings, R. (1972). *Mathematics in the Time of the Pharaohs*, pp. 139-146. Dover, New York.

Heath, T. (1921). *A History of Greek Mathematics: Volume I, From Thales to Euclid*, pp. 220-235. Oxford University Press, London.

Hobson, E. (1913). *Squaring the Circle: A History of the Problem*, pp. 16-22. Cambridge University Press.

Navarro, J. (2010). *The Secrets of pi: Why the circle can't be squared*, pp. 16-22 & 60-66. RBA Coleccionables.

Sarasvati, A. (1999). *Geometry in Ancient and Medieval India*, pp. 34-36. Motilal Banarsidass, Delhi.

Schubert, H. (1891). The Squaring of the Circle. An Historical Sketch of the Problem from the Earliest Times to the Present Day. In P. Carus (Ed.), *The Monist*, *1*(2), pp. 197-228. Retrieved from www.jstor.org/stable/27896849. Oxford University Press.

Scriba, C. & Schreiber, P. (2015). *5000 Years of Geometry: Mathematics in History and Culture*, pp. 52-53 & pp. 158-159. Springer, New York.

Seidenberg, A. (1962). The Ritual Origin of Geometry, in *Archive for History of Exact Sciences 1*(5), pp. 515-517. Springer.

Wilson, R. (2008). *Squaring the Circle and other Impossibilities*, (lecture). Retrieved from http://bit.ly/SquaringTheCircleImpossible

RESOURCE

www.bit.ly/squaring-the-circle Free downloads, beach maths ideas and photos, ask questions and share your circle squaring experiences. Hashtag #WESTC (WE Squared The Circle!)